# DESIGN STUDIES OF COHERENT PREBUNCHING AND EMITTANCE REDUCTION FOR THE MaRIE XFEL*

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*Abstract*

There are several schemes currently being investigated which use modulator and dispersive sections to step the coherent bunching of the electron beam up to higher harmonics [1-4]. X-ray FELs generally operate in a regime where the FEL parameter, ρ is equal to or less than the effective energy spread introduced from the emittance in the electron beam. Because of this large effective energy spread, the energy modulation introduced from harmonic generation schemes would seriously degrade FEL performance. This problem can be mitigated by incorporating the harmonic generation scheme at a lower electron kinetic energy than the energy at the final undulator. This will help because the effective energy spread from emittance is reduced at lower energies, and can be further reduced by making the beam transversely large. Then the beam can be squeezed down slowly enough in the subsequent accelerator sections so that geometric debunching is avoided. Here we show analytical results that demonstrate the feasibility of this harmonic pre-bunching scheme.

## INTRODUCTION

The effective energy spread introduced from the emittance of a relativistic electron beam is given by [5]:

This will lead to a spread in electron velocities that tends to damp out electron bunching. The effective energy spread is typically large (comparable to ρ or greater) for a hard x-ray FEL, and hence will increase the FEL gain length. Harmonic generation schemes involve introducing an additional energy modulation ~2x the random energy spread in the beam [1-4], which can seriously degrade FEL performance.

We are currently exploring different options for mitigating this emittance effect by decreasing the effective energy spread in the harmonic generation sections. Eqn. (1) shows that this energy spread can be decreased by either increasing the transverse size of the beam, or having harmonic generation located at lower electron energies than the final undulator, and attempting to preserve the electron bunching through accelerator sections. Both of these possibilities have limitations: making the transverse size large will increase the gain length [5] and degrade the transverse coherence of the x-rays [6], while staging harmonic generation at low energies is limited because the linear $R_{56}$ effect from the accelerator sections soon becomes too large and causes overbunching. However, if this energy spread can be reduced in the harmonic gain sections, then electron harmonic bunching can be achieved with lower energy modulations introduced to the beam, thus reducing the gain length in the final undulator.

Here we will explore the feasibility of this scheme for the proposed MaRIE XFEL at Los Alamos National Laboratory. The MaRIE XFEL must generate ¼ Å, longitudinally coherent x-rays with a 20 GeV electron beam. Techniques are being explored for generating coherent bunching at sub-harmonics of the final radiation [10]. Here, we will examine only the final stage of harmonic generation, where the coherent bunching is stepped up from 1 Å to ¼ Å.

## DESIGN

The design we will explore is shown in fig. 1. The electron beam, which is bunched at 1 Å, has an energy of 17.5 GeV and a transverse waist size of 21 μm. This beam passes through a modulator which is resonant at 1 Å. The beam is transversely large to mitigate the emittance debunching effect of Eqn. 1. The bunching will radiate, which in turn will generate an energy modulation at 1 Å. After this, the beam will pass through a chicane. In the chicane, there is linear dispersion $R_{56}$, which will sharpen the sinusoidal pulse into a shock which has significant bunching at high harmonics. Then the beam is slowly squeezed down transversely to 10 μm while in an accelerator section, so that the current density in the final undulator will be large enough to generate ¼ Å radiation.

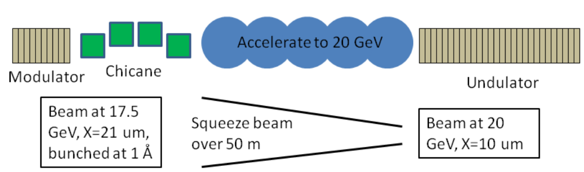


Figure 1: Layout of the scheme described in this paper.

Some or all of the linear dispersion can come from the accelerating section in between the two wigglers. This in fact is the lower energy limit of the 1 Å modulator section; below that energy the $R_{56}$ from the drift becomes too large, and the beam becomes over -bunched.

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For this design we will assume a normalized emittance of 0.15 μm in the transverse directions. This is smaller by a factor of 2 than the normalized slice emittance in LCLS [7]. Techniques are being explored to achieve this lower emittance through the eigen-emittance concept [8].

### *Nonlinear Debunching in Accelerator Section*

In the acceleration section after the harmonic generation section the beam will debunch from nonlinear effects from the emittance of the beam. The RMS value of the angular spread of the beam is given by:

$$\langle x'^2 \rangle = \frac{\varepsilon_n^2}{\gamma^2 \boldsymbol{X}^2} + (\boldsymbol{X}')^2, \quad \boldsymbol{X}^2 = \langle x^2 \rangle \tag{1}$$

Here $\varepsilon_N$ is the normalized emittance of the beam, and $\boldsymbol{X}' = d\boldsymbol{X}/ds$ is spatial derivative of the size of the bunch. The difference in path length of a beam with some angular divergence is given by $\frac{dz}{ds} \approx 1 - \frac{(x')^2}{2} - \frac{(y')^2}{2}$. Then the RMS particle debunching from an accelerator section assuming a round beam and constant acceleration will be given by:

$$\delta z = \left[ \int_{s_0}^{s_f} \frac{\varepsilon_n^2}{(\gamma_0 + \gamma' s)^2 \boldsymbol{X}^2} + (\boldsymbol{X}')^2 \right] ds \tag{3}$$

Here $\gamma'$ is the acceleration gradient and $s$ denotes the length along the accelerator section. There is a trade-off between the debunching caused by emittance, and the debunching caused by geometric effects of changing the beam size. In addition, the beam must become small enough at the final undulator so that the FEL will work properly.

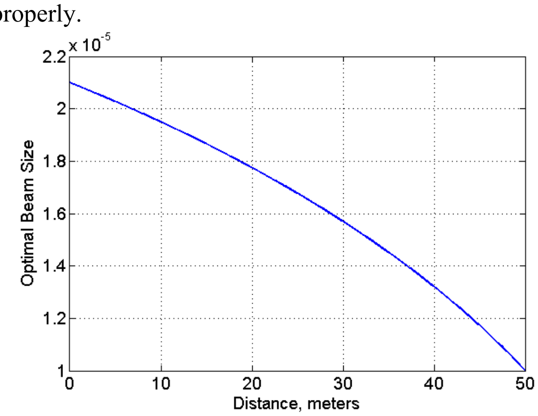


Figure 2: The optimal beam size for minimizing debunching in an accelerator section with gradient 50 MV/m, accelerating the beam from 17.5 GeV to 20 GeV.

Using the calculus of variations [10], the minimized debunching along the accelerator will happen when the beam size follows a path given by:

$$\boldsymbol{X}'' = \frac{-\varepsilon_n^2}{(\gamma_0 + \gamma' s)^2 \boldsymbol{X}^3} \tag{4}$$

This optimized beam size is calculated numerically. Given an acceleration gradient, an acceleration length, and a desired initial and final transverse beam size, there is a unique optimized solution. Fig. 2 shows a plot of the optimized transverse size for minimizing debunching in an accelerator section with gradient 50 MV/m, accelerating the beam from 17.5 GeV to 20 GeV and with an initial and final transverse size of 21 μm and 10 μm.

For the purpose of calculating harmonic generation, we will describe the deviation in path length with $\delta\theta$, where $\delta\theta = 2\pi\delta z/\lambda$ and $\lambda$ = 1 Å is the sub-harmonic wavelength. We can convert the nonlinear debunching into an effective induced energy spread using: $\delta\theta_{NL} = \delta\gamma_{NL}(\frac{d\theta}{d\gamma})$, where $\frac{d\theta}{d\gamma}$ is the linear dispersion.

### *Linear Dispersion in Accelerator Section*

Electrons will also experience linear dispersion in the accelerator section. This will act like a chicane, and lead to harmonic generation in a modulated beam. However, if it becomes too large there will be overbunching and the harmonic generation will be damped.

Integrating along an accelerator section, the linear dispersion becomes, for ultra-relativistic beams:

$$\frac{dz}{d\gamma} = \frac{1}{2\gamma'} \left( \frac{1}{\gamma_0^2} - \frac{1}{\gamma_f^2} \right) \tag{5}$$

Here $\gamma_0$ and $\gamma_f$ are the initial and final energies, respectively.

The linear dispersion becomes too large if we try to modulate the beam at energies below 17 GeV.

### *Nonlinear Debunching in Chicane*

The chicane will introduce an additional linear dispersion. Because the chicane will be small, the size of the beam will not change in the chicane. Thus we can calculate the nonlinear debunching in the chicane by calculating the induced energy spread from Eqn. 1.
The effect of the chicane and the effect of a drift section are essentially the same. Both introduce some linear dispersion, and some nonlinear energy spread given by Eqn. 1. The linear dispersion decays more rapidly with $\gamma$ than the nonlinear debunching, both in a magnet and a drift section. Thus, at lower energies a smaller total modulation energy is needed to overcome the nonlinear debunching.

### *Total Energy Spread and Linear Dispersion*

The total linear dispersion will be the sum of the dispersion from the chicane and the dispersion from the accelerator section,

$d\theta/d\gamma = (d\theta/d\gamma)_{chicane} + (d\theta/d\gamma)_{accel}$. The total energy spread will be the intrinsic energy spread, $\sigma_\gamma$ and the nonlinear energy spread, added in quadrature because they are uncorrelated. Because the energy spread from nonlinearity in the chicane and accelerator come from the same source, they are added directly.

$$\sigma_{\gamma,tot} = \sqrt{{\sigma_\gamma}^2 + \left( \frac{\delta\theta_{NL}}{d\theta/d\gamma} + \frac{\gamma_0 \epsilon_N^2}{\langle x^2 \rangle} \right)^2}$$

This can be used to estimate the total harmonic bunching produced from a modulator-dispersive section followed by an acceleration section.

### *Estimation of Harmonic Bunching*

In our scheme, the beam is modulated at the sub-harmonic, and then put through a dispersive section. Assuming that the initial spatial bunching at 1 Å is small enough to be ignored, this will have the same kinetic properties as a High Gain Harmonic Generation (HGHG) section [1, 2]. The bunching factor from standard HGHG is [2]:

$$b_n = 2\exp\left[-\frac{1}{2}n^2{\sigma_\gamma}^2\left(\frac{d\theta}{d\gamma}\right)^2\right]\mathrm{J_n}\left(\mathrm{n}\Delta\gamma\frac{d\theta}{d\gamma}\right)$$

Here $\sigma_\gamma$ is the intrinsic energy spread of the beam, n is the harmonic number, and $\Delta\gamma$ is the modulation energy. If we include nonlinear effects, as well as debunching and additional dispersion in the accelerator section, the bunching factor becomes:

$$b_n = 2\exp\left\{-\frac{1}{2}n^2\left[{\sigma_\gamma}^2 + \left(\frac{\delta\theta_{NL}}{d\theta/d\gamma} + \frac{\gamma_0\epsilon_N^2}{\langle x^2\rangle}\right)^2\right]\left(\frac{d\theta}{d\gamma}\right)^2\right\}\mathrm{J_n}\left(\mathrm{n}\Delta\gamma\frac{d\theta}{d\gamma}\right) \quad (6)$$

Here $\delta\theta_{NL}$ is the nonlinear debunching in the accelerator section, and $\frac{d\theta}{d\gamma}$ is the total linear dispersion.

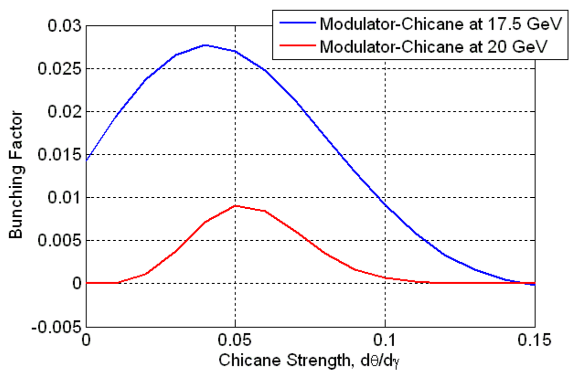


Figure 3: Bunching factor at final undulator, including nonlinear debunching effects, with (Blue) Modulator-chicane located 50 m before final undulator, at 17.5 GeV (Red) Modulator-chicane located right before final undulator, at 20 GeV.

We have developed a code to look at the effects of placing the modulator-chicane section at lower beam energies and larger transverse sizes. The code first calculates the total nonlinear debunching, $\delta\theta_{NL}$, for a given starting energy, starting transverse size and final transverse size, using eqns. 3 and 4. The linear dispersion in the acceleration section is also calculated from eqn. 5. The acceleration gradient is assumed to be 50 MV/m, the final energy is 20 GeV, and the fundamental harmonic is 1 Å. The code then uses eqn. (6) to calculate the bunching at the 4$^{th}$ harmonic (1/4 Å), for different modulation energies $\Delta\gamma$ and chicane strengths. Here the intrinsic energy spread is assumed to be $\sigma_\gamma = 2$.

The modulation strength is controlled by adjusting the length of the initial modulator section. Because the modulation is at a sub-harmonic of the final x-ray frequency, it acts like an additional energy spread in the beam in the final undulator [1], and must be kept small. We allow a modulation strength of $\Delta\gamma = 9$; slightly less than the FEL parameter ρ of the final undulator.

Figure 3 shows the results of these calculations. The blue curve corresponds to the situation shown in fig. 1, where the modulator-chicane is located 50 m before final undulator, at 17.5 GeV, and the beam is squeezed from 21 μm to 10 μm in the accelerator section. The red curve corresponds to the situation where the modulator-chicane section is located right before the final undulator, where the electron beam has been accelerated to 20 GeV and the transverse size of the electron beam is 10 μm. The bunching is 3 times greater when the modulator-chicane is moved back 50 m. In both cases, the bunching is significantly reduced when nonlinear effects are included.

## CONCLUSION

We have demonstrated a technique for reducing the nonlinear debunching effects from emittance in a harmonic generation scheme. We have shown the effects for the final stage, going from 1 Å to ¼ Å. In order to produce coherent bunching at 1 Å, a multi-stage harmonic generation approach will be needed [11]. For stages before the final one, the beam will already be large, and the emittance effect will be negligible. This will significantly reduce the total energy spread that must be introduced to the electron beam from the modulators in order to produce adequate electron bunching.

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